New Limit Cycle Bifurcations in 3D Piecewise Linear Systems

Enrique Ponce, Francisco Torres
Dpto Matematica Aplicada II Universidad de Sevilla.

There exist very few results that rigorously prove the existence of limit cycles in three-dimensional systems. In order to study the existence of such periodic orbits perturbative methods and the averaging methodology are mainly used. However, it remains to solve the very important issue of determining the oscillatory behavior of systems with three or more variables by using algebraic or analytic methods. Once established the existence of periodic orbits, this situation could become as the starting point of other techniques, such perturbative ones, to deal with a broader class of systems.

This talk is devoted to show the existence of limit cycles in piecewise linear three-dimensional systems. More precisely, we will investigate the system

$$\dot{\mathbf{x}} = \begin{cases} A\mathbf{x} + \mathbf{b}, & \text{if} \quad x > 1, \\ B\mathbf{x}, & \text{if} \quad |x| \leqslant 1, \\ A\mathbf{x} - \mathbf{b}, & \text{if} \quad x < -1, \end{cases} \tag{1}$$

where $\mathbf{b} = (6\alpha - \epsilon, 1 - 11\alpha^2 - \epsilon^2, 6\alpha^3 + \epsilon + \epsilon^3)^T$, $\mathbf{x} = (x, y, z)^T$,

$$A = \begin{pmatrix} -6\alpha & -1 & 0 \\ 11\alpha^2 & 0 & -1 \\ -6\alpha^3 & 0 & 0 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} -\epsilon & -1 & 0 \\ 1 - \epsilon^2 & 0 & -1 \\ \epsilon + \epsilon^3 & 0 & 0 \end{pmatrix}.$$

The matrix A has three real negative eigenvalues, namely $-\alpha$, -2α and -3α, while the central matrix B has the eigenvalues $\epsilon \pm i$ and ϵ. When $\epsilon = 0$, the system has a bounded set completely foliated by periodic orbits which has the shape of two solid cones sharing the disc $x^2 + y^2 \leqslant 1$ in the plane $z = 0$ as their common basis. Here, we state a first result,

Theorem 1 *We consider system (1) with $\alpha > 0$ and $\epsilon = 0$. For $\alpha = 1$ the system undergoes a limit cycle bifurcation. The limit cycle bifurcates from the circle $\{x^2 + y^2 = 1,\ z = 0\}$ and persists for all $\alpha > 1$, being hyperbolic and stable. Moreover, the limit cycle passes through the points $P_0 = (1, y_0, z_0)$, $P_1 = (1, y_1, z_1)$, $P_2 = (-1, -y_0, -z_0)$ and $P_3 = (-1, -y_1, -z_1)$, and it is possible to give analytical expressions, depending on the parameter α, for the period and its intersection points with the planes $x = \pm 1$.*

When $\epsilon \neq 0$ and small, it can be analytically shown the existence of another periodic orbit which is born from one of the most external periodic orbits of the bounded continuum of periodic orbits existing for $\epsilon = 0$, so giving rise to the coexistence of three periodic orbits, due to the symmetry of the vector field.

Coalescing of singular values for matrices that depend on parameters

Alessandro Pugliese
Dipartimento di Matematica, Università di Bari

It is well understood that there are bifurcations phenomena for systems depending on several parameters which cannot be observed, generically, by by freezing one parameter at a time. One notable example is coalescing of singular values for real matrices that depend smoothly on two parameters. In this talk we review the main features of this phenomenon. In particular: (i) we relate the existence of points where two singular values coalesce to the period doubling of the orthogonal factors of the smooth SVD along loops containing such points, (ii) we exploit this period doubling to construct numerical algorithms aimed at detecting and accurately approximating the coalescing points, (iii) we discuss how the detection of coalescing points could help identifying regions in parameters' space where certain dynamical systems exhibit hypersensitive behavior.

This is joint work with Prof. Luca Dieci (School of Mathematics, Georgia Institute of Technology)

Complete Bifurcation Analysis of Strongly Nonlinear Driven Systems with Several-degree-of-freedom with Several Equilibrium Positions

M.V. Zakrzhevsky[1a], I.T. Schukin[2b], V.Yu. Yevstignejev[1c], R.S. Smirnova[2], A.V. Klokov[1]
[1] Institute of Mechanics, Riga Technical University
1 Kalku Str., Riga, LV-1658, Latvia
[2] Daugavpils branch of Riga Technical University
90 Smilshu Str., Daugavpils, LV-5410, Latvia
E-mail: [a]*mzakr@latnet.lv*, [b]*igor@df.rtu.lv*, [c]*vljevst@latnet.lv*

This paper devoted to application of the new method of complete bifurcation groups (MCBG) [1], which shows very good results in single-degree-of-freedom tasks, for numerical global bifurcation analysis of complex nonlinear dynamical driven and autonomous systems with several-degree-of-freedom with several equilibrium positions [2-4]. Models under consideration are: driven body in asymmetrical elastic field; driven double pendulum; driven two-mass chain system with symmetrical elastic characteristic with two potential wells between masses; driven and autonomous two-mass chain linear system with nonmonotonic dissipative characteristic between masses; driven six-mass chain system with nonlinear elastic characteristics between masses. In the paper authors discuss advantages of complete bifurcation analysis and received on base of the MCBG new or little-studied kinds of bifurcations and bifurcation groups, such as rare attractors, complex protuberances, unstable periodic infinitiums, and “desert” isles. Some results are compared with exact analytical solutions.

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