Equivariant dynamical systems in infinite dimensions

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ABSTRACT

Time dependent partial differential equations may generally be viewed as dynamical systems in infinite dimensional (function-) spaces. This leads to a vast area of applications for the numerical analysis of dynamical systems. A new issue that becomes crucial is spatial discretization which replaces the infinite dimensional by a finite dimensional phase space. It has to be combined with traditional methods for studying the dynamics such as simulation and the computation of equilibria, periodic orbits, and their bifurcations.

In our contribution we concentrate on infinite dimensional dynamical systems that have an extra continuous symmetry. More specifically we consider vector fields on function spaces that are equivariant with respect to the action of a (finite dimensional) Lie group. Typical examples are provided by (autonomous) reaction-diffusion systems and the Navier-Stokes equations on the whole space which are equivariant with respect to the action of the Euclidean group.

We consider numerical methods for such equations, in particular the use of equivariance when studying relative equilibria, relative periodic orbits and their stability and bifurcations. For the Cauchy problem we discuss the *freezing method* that uses equivariance to separate shape dynamics from group dynamics. During simulation a comoving frame is computed in which a moving pattern that converges to a relative equilibrium become stationary. We discuss applications to various types of nonlinear waves (traveling, rotating, spiral, scroll) that occur in excitable reaction diffusion systems.