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"Towards a qualitative bifurcation theory for piecewise-smooth dynamical systems"

Many dynamical systems that occur naturally in the description of physical processes are piecewise-smooth. That is, their motion is charcterised by periods of smooth evolutions interrupted by instantaneous events. Traditional analysis of dynamical systems has restricted its attention to smooth problems, thus preventing the investigation of nonsmooth processes such as impact, switching, sliding and other discrete state transitions. These phenomena arise in many applications for example anything involving friction, collision, intermittently constrained systems or processes with switching components.

The purpose of this talk is to introduce a similar qualitative theory for nonsmooth systems similar to that which exists for smooth dynamical systems. In particular we shall propose general techniques for analyzing the bifurcations that are unique to nonsmooth dynamical systems, so-called *discontinuity-induced* bifurcations (DIBs for short). This we propose as a general term for all transitions in dynamics specifically brought about through interaction of invariant sets of the system ('attractors') with a boundary in phase space across which the system has some kind of discontinity. First and foremost, we shall give a consistent classification of all known DIBs for piecewise-smooth continuous-time dynamical systems (flows), including such diverse phenomena as sliding, chattering, grazing and corner collision. We will then describe a unified analytical framework for reducing the analysis of each such bifurcation involving a periodic orbit to that of an appropriately-defined Poincaré map. This process is based on the construction of so-called *discontinuity mappings* which are analytical corrections made to account for crossing or tangency with discontinuity boundaries. We introduce the notion of the *degree of smoothness* depending on whether the state, the vector field or one of its derivatives has a jump across a discontinuity boundary. We show how standard examples such as impact oscillators, friction systems and relay controllers can be put into this framework, and show how to construct discontinuity mappings for tangency of each kind of system with a discontinuity boundary.