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Numerical Continuation Methods with Application to Computing Stable/Unstable Manifolds

Abstract:

We first describe the general setting of equations that give rise to (one-dimensional) *solution families* and we show how Keller's continuation method can be used to compute such families. As a simple introductory example we compute stationary solution families of the Lorenz equations. We then consider equations consisting of differential equations subject to boundary and integral constraints, and we indicate their use in computing families of periodic solutions of dynamical systems modeled by ordinary differential equations. As our main application of numerical continuation we show the remarkable effectiveness of boundary value formulations for computing stable and unstable manifolds. Our first example concerns the so-called Lorenz manifold, for which the computations provide insight into the nature of the Lorenz attractor. Our second manifold example concerns the Circular Restricted Three-Body Problem, which models the motion of a satellite in an Earth-Moon-like system. Specifically we compute the unstable manifold of periodic orbits known as *Halo orbits*, which have been used in actual space missions. Our calculations lead to the detection of heteroclinic connections from a periodic Halo orbit to invariant tori. Subsequent continuation of such connections (as the Halo orbit is allowed to change) leads to a variety of connecting orbits that may be of interest in space-mission design.